**SUMMARY/ABSTRACT**

The United States’ penny coin has different composition year by year and minted at different locations. In 1982, the original coin’s composition changed within the same year, causing the mass change, so it is analyzed by separating the data sets into two different groups: pre-1982.0 and post-1982.5. Statistical tests such as F-test and Student’s t-test are used to quantitatively justify each data sets’ behavior.

**INTRODUCTION**

The objective of this lab is to study a set of given data and apply various statistical tests on it. The United States one-cent coins, mainly known as penny, are used as the subject of this experiment. The masses of the pennies are collected from classes in the past and are analyzed in this lab. It is being used due to its mass-produced, inexpensive mass and convenience properties. The masses of the pennies have different compositions by year. They were also minted at different locations (Philadelphia, Denver, San Francisco and West Point) and labelled differently (P, D, S and W respectively). The pennies produced in 1982 can be separated easily by mass: the heavier pennies is put into pre-1982.0 group while the lighter pennies is put into post-1982.5 group.

**EXPERIMENTAL PROCEDURE**

First, the given set of data is sorted into the two said groups. From there, the outliers are removed from each group (± 3σ, standard deviation). The steps are repeated up to the point where there are no more outliers left. At every single step of outlier removal, the average, standard deviation, minimum, maximum and relative standard deviation in parts-per-thousands of masses of pennies are calculated. The final data from both groups are used for the rest of the analysis. Each year’s average is calculated and a graph of mass of penny versus the year is plotted, with an error bar equal to plus or minus of one standard deviation.

Year-by-year analysis by mass is done on the post-1982.5 group first. Pennies from 2009 are removed beforehand. A graph of mass of penny versus the year is plotted and the slope of the linear trend line is compared with its σ. At 95% confidence, the slope is determined by using Student’s t-test, whether it is negative, positive or zero. After the first analysis, the highest and lowest average mass of those year are compared at the 95% confidence level, to determine whether they are significantly different or not. All of these steps are repeated for the pre-1982.0 group, without removing the pennies from 2009 as it is not part of the group.

Analysis on comparison of mint locations are done next. By using the same data sets of removed outliers, the groups are then being sorted based on their minted location. For the group of post-1982.5, pennies from 2009 is again removed, and the average, standard deviation, minimum and maximum for the mass of each location are calculated. The average mass of pennies minted at different mints location are compared with each other by using Student’s t-test at 95% confidence level. These steps are repeated for the pre-1982.0 group.

The last analysis involved the study of mass distribution of post-1982.5 group, excluding pennies from 2009. By using the removed outliers set of data, the average, standard deviation, minimum and maximum mass of pennies of each year are determined. A histogram is generated from the set of data, ranging from 2.4 g to 2.6 g, with an interval of 0.01 g.

**RESULTS AND DISCUSSION**

***Removal of outliers***

Pre-1982.0 group’s outliers were removed for 4 times while the post-1982.5 group’s outliers were removed for 5 times. The average, standard deviation, minimum, maximum and relative standard deviation in PPT of each group are tabulated in **Table 1** and **2** respectively. The final set of data of these values are as shown in **Table 3**. The formula used to determine these calculated values are as listed below:

* Mean = AVERAGE(range of values)
* Standard deviation = STDEV(range of values)
* ± 3 σ = ± 3 x Standard deviation
* Minimum value = MIN(range of values)
* Maximum value = MAX(range of values)
* Relative standard deviation in PPT = (Standard deviation / Mean) x 1000

***Year-by-year analysis***

Average mass of penny for every year and its respective standard deviation (σ) are tabulated in **Table 4A** and **4B**. A graph of average mass of penny versus its year are plotted, with an error bar of ± σ for each year’s average, as shown in **Figure 1**. The data were separated into two groups, but they were plotted in one graph.

***Year-by-year analysis for post-1982.5 and pre-1982.0***

A mass of penny versus its year of both groups are plotted, excluding the mass of pennies from 2009 for the post-1982.5 group as shown in **Figure 2** and **3** respectively, with their trend line, intercept and correlation coefficient R value included in the plot. The standard deviation of the mean, σm, for both groups are determined by using the LINEST function in excel. The formula is as shown below:

**LINEST function = LINEST(y-axis value, x-axis value, 1,1)**

The formula shown above will give all of the following data: mean (m), standard deviation of the mean (μm or σm), correlation coefficient R (R2), Fisher value, regression sum of squares, y-intercept (b), standard error of the y-intercept (μb), standard deviation in y-values (sy), degree of freedom (df) and residual sum of squares. These values are tabulated in **Table 5** and **8** respectively.

The slopes of both groups are compared with their error σm.For both groups**,** the historical trend in the mass of pennies are to be downward sloping, meaning that the mass of pennies decreases by year. The data are calculated as shown in **Table 6** and **9** respectively. Both of the slopes of the trend line (the rate at which the mass changing by year (g/year), with a ± σm on the slope m) come out as negative values, indicating that the mass decreases by year for both groups.

By using a Student’s t-test with a 95% confidence interval, as shown in **Table 7** and **10** respectively, with a ± t\*μm, both slopes of the trend line still come out as negative values. We are 95 % confident that there is a decreasing trend in mass of the pennies each year for both groups.

The highest and lowest average masses of pennies in each group are determined, as shown in **Table 11** and **14** respectively. For the post-1982.5 group, an F-test is first conducted in order to determine whether the standard deviation of both the highest and lowest average masses of pennies come from the same population standard deviation or not, before running the Student’s t-test, to determine the difference in average mass between these two is significant or insignificant. By referring to **Table 12**, these set of data passed the F-test, meaning that the F-calculated value is lower than the F-table value. It can be concluded that these set of data come from a population with the same standard deviation.

The t-test is then conducted, and the values are calculated and tabulated as shown in **Table 13**. The t-calculated value is bigger than the t-table value, indicating that it fails the t-test. It can be concluded that the difference between these average masses are significant.

The same thing is done on the pre-1982.0 group. The highest and lowest average mass are determined and tabulated in **Table 14**. An F-test is then conducted on this group as shown in **Table 15**. It shows that the F-calculated value is higher than the F-table value, in which it fails the F-test. The two sets of data do not come from a population of the same standard deviation.

The t-test is further conducted on this set of data and tabulated in **Table 16**. It is found that the t-calculated value is larger than the t-table value, indicating that the difference in these average masses are significant.

In order to determine the highest and lowest average mass for each group, these formula are applied: Highest average mass = MAX(range of values); Lowest average mass = MIN(range of values).

Both F-test and t-test are conducted by using the add-ins tool in the Excel, under the Data Analysis starter pack. A t-test with equal variance is used for a set of data that passed the F-test, and the t-test with unequal variance is used for a set of data that failed the F-test.

***Comparison by mint locations***

This analysis is done again on both groups of data. This time, the data were sorted by the pennies’ minted locations. After the outliers are removed on both groups, only the pennies minted at Philadelphia (P) and Denver (D) are left to be analyzed. For each group, the average, standard deviation, minimum and maximum value of the mass of pennies are determined and they are tabulated in **Table 17** and **20** for post-1982.5 and pre-1982.0 respectively.

The average mass of pennies minted at different locations for each group is compared with each other by conducting the Student’s t-test. First, an F-test is run on these data. As shown in **Table 18**, the results of F-test for post-1982.5 group shows that it passed the F-test (F-calculated value is smaller than the F-table value), indicating that the data sets indeed came from the population of the same standard deviation.

It is further analyzed by running the t-test on these data, and the results are tabulated in **Table 19**. As shown, it passed the t-test, when t-calculated is smaller than t-table value. The difference in average masses between these two different minted locations are insignificant.

These analyses are then repeated on the pre-1982.0 group. The F-test result of this group is tabulated in **Table 21**. By referring to that same table, it is found that the data sets failed the F-test, as the F-calculated value is greater than the F-table value, meaning that these data sets came from a population with different standard deviation.

The t-test is then conducted on this data set and the results are as shown in **Table 22**. The difference in the average masses between these two different minted locations are found to be insignificant, as the t-calculated value is smaller than the t-table value (pass the t-test). Both of these tests are conducted the same way as explained in the year-by-year analysis part.

***Distribution of masses***

This part of analysis only involved the data in the post-1982.5 group, by excluding the 2009’s data. The average, standard deviation, minimum and maximum value of penny mass of each year are determined and tabulated in **Table 23**. The formula for these data measurement is the same as explained in removal-of-outliers analysis part. A histogram is generated for this data and as shown **Figure 4**. The frequency of each range of masses of pennies are generated for this histogram and is shown in **Table 24**.

**CONCLUSIONS**

In a nutshell, this lab helps students to familiarize themselves with the Excel. Students are also able to quantitatively justify certain data sets by using various statistical tests: F-test and Student’s t-test. It is said to pass the F-test if the F-calculated is lower than the F-table, in which indicates that the data sets come from the same standard deviation population, and vice versa for failed F-test. Meanwhile, it is said to pass the t-test if the t-calculated is lower than the t-table, which indicates that the two given means do not differ significantly, and vice versa with the failed t-test.

**TABULATED AND PLOTTED DATA PORTION OF THE ANALYSIS**

**REMOVAL OF OUTLIERS ANALYSIS**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Pre-1982.0** | | | | |
| **Number of outlier removal** | Original | 1 | 2 | 3 | 4 |
| **Mean (g)** | 3.0837 | 3.0891 | 3.0892 | 3.0892 | 3.0892 |
| **Standard deviation (g)** | 0.0867 | 0.0334 | 0.0306 | 0.0301 | 0.0300 |
| **Minimum value (g)** | 2.0152 | 2.9276 | 2.9935 | 2.9978 | 2.9997 |
| **Maximum value (g)** | 3.9652 | 3.2625 | 3.1849 | 3.1749 | 3.1749 |
| **Relative standard deviation (ppt)** | 28.1043 | 10.8246 | 9.9181 | 9.7336 | 9.7122 |
| **3 σ (g)** | 0.2600 | 0.1003 | 0.0919 | 0.0902 | 0.0900 |
| **Mean + 3 σ (g)** | 3.3437 | 3.1894 | 3.1811 | 3.1794 | 3.1792 |
| **Mean - 3 σ (g)** | 2.8237 | 2.9888 | 2.9973 | 2.9990 | 2.9992 |

**Table 1:** Outliers removal of pre-1982.0 group

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **Post-1982.5** | | | | | |
| **Number of outlier removal** | Original | 1 | 2 | 3 | 4 | 5 |
| **Mean (g)** | 2.5077 | 2.5053 | 2.5048 | 2.5046 | 2.5046 | 2.5046 |
| **Standard deviation (g)** | 0.0431 | 0.0210 | 0.0195 | 0.0193 | 0.0191 | 0.0191 |
| **Minimum value (g)** | 2.4083 | 2.4083 | 2.4467 | 2.4467 | 2.4480 | 2.4480 |
| **Maximum value (g)** | 3.5135 | 2.5976 | 2.5656 | 2.5629 | 2.5621 | 2.5616 |
| **Relative standard deviation (ppt)** | 17.2073 | 8.3958 | 7.7665 | 7.6899 | 7.6317 | 7.6174 |
| **3 σ (g)** | 0.1294 | 0.0631 | 0.0584 | 0.0578 | 0.0573 | 0.0572 |
| **Mean + 3 σ (g)** | 2.6371 | 2.5684 | 2.5631 | 2.5624 | 2.5619 | 2.5618 |
| **Mean - 3 σ (g)** | 2.3782 | 2.4422 | 2.4464 | 2.4469 | 2.4472 | 2.4473 |

**Table 2:** Outliers removal of post-1982.5 group

|  |  |  |
| --- | --- | --- |
|  | **Final removal of outliers** | |
| **Group** | Pre-1982.0 | Post-1982.5 |
| **Mean (g)** | 3.0892 | 2.5046 |
| **Standard deviation (g)** | 0.0300 | 0.0191 |
| **Minimum value (g)** | 2.9997 | 2.4480 |
| **Maximum value (g)** | 3.1749 | 2.5616 |
| **Relative standard deviation (ppt)** | 9.7122 | 7.6174 |
| **3 σ (g)** | 0.0900 | 0.0572 |
| **Mean + 3 σ (g)** | 3.1792 | 2.5618 |
| **Mean - 3 σ (g)** | 2.9992 | 2.4473 |

**Table 3:** Final removal of outliers for both group

**YEAR-BY-YEAR ANALYSIS**

|  |  |  |
| --- | --- | --- |
| **Pre-1982.0** | | |
| **Year** | **Mean (g)** | **Standard deviation (g)** |
| **1950** | 3.0881 | 0.0000 |
| **1956** | 3.0883 | 0.0001 |
| **1959** | 3.1056 | 0.0168 |
| **1960** | 3.0994 | 0.0000 |
| **1961** | 3.0957 | 0.0230 |
| **1962** | 3.1035 | 0.0333 |
| **1963** | 3.1003 | 0.0453 |
| **1964** | 3.0980 | 0.0366 |
| **1965** | 3.0926 | 0.0211 |
| **1966** | 3.0942 | 0.0166 |
| **1967** | 3.0848 | 0.0265 |
| **1968** | 3.0707 | 0.0281 |
| **1969** | 3.0936 | 0.0341 |
| **1970** | 3.1021 | 0.0358 |
| **1971** | 3.1070 | 0.0429 |
| **1972** | 3.0820 | 0.0209 |
| **1973** | 3.0755 | 0.0305 |
| **1974** | 3.0889 | 0.0368 |
| **1975** | 3.0967 | 0.0241 |
| **1976** | 3.0876 | 0.0296 |
| **1977** | 3.0833 | 0.0297 |
| **1978** | 3.0920 | 0.0263 |
| **1979** | 3.0846 | 0.0265 |
| **1980** | 3.0876 | 0.0267 |
| **1981** | 3.0923 | 0.0266 |
| **1982.0** | 3.0839 | 0.0266 |

|  |  |  |
| --- | --- | --- |
| **Post-1982.5** | | |
| **Year** | **Mean (g)** | **Standard deviation (g)** |
| **1982.5** | 2.5149 | 0.0244 |
| **1983** | 2.5189 | 0.0253 |
| **1984** | 2.5170 | 0.0210 |
| **1985** | 2.5246 | 0.0192 |
| **1986** | 2.5192 | 0.0238 |
| **1987** | 2.5031 | 0.0199 |
| **1988** | 2.5011 | 0.0217 |
| **1989** | 2.5179 | 0.0190 |
| **1990** | 2.5076 | 0.0203 |
| **1991** | 2.5053 | 0.0204 |
| **1992** | 2.5016 | 0.0235 |
| **1993** | 2.5010 | 0.0152 |
| **1994** | 2.5027 | 0.0124 |
| **1995** | 2.4975 | 0.0142 |
| **1996** | 2.5016 | 0.0186 |
| **1997** | 2.4924 | 0.0173 |
| **1998** | 2.5024 | 0.0138 |
| **1999** | 2.5019 | 0.0141 |
| **2000** | 2.5022 | 0.0111 |
| **2001** | 2.5040 | 0.0144 |
| **2002** | 2.4996 | 0.0112 |
| **2003** | 2.4983 | 0.0171 |
| **2004** | 2.5016 | 0.0136 |
| **2005** | 2.5023 | 0.0197 |
| **2006** | 2.4993 | 0.0183 |
| **2007** | 2.4984 | 0.0148 |
| **2008** | 2.5060 | 0.0166 |
| **2009** | 2.4961 | 0.0162 |
| **2010** | 2.4978 | 0.0140 |
| **2012** | 2.5155 | 0.0000 |
| **2013** | 2.4938 | 0.0100 |

**Table 4A & 4B:** Average mass of pennies each year, for pre-1982.0 and post-1982.5 group respectively

**Figure 1:** Graph of US One Cent coins from 1950 to the present year

**Figure 2:** US One Cent coins from 1982.5 to 2013, excluding 2009

|  |  |  |  |
| --- | --- | --- | --- |
| **LINEST POST-1982.5** | | | |
| **m** | -0.000671 | 3.844721 | **b** |
| **μm** | 0.000051 | 0.101141 | **μb** |
| **R2** | 0.076542 | 0.018349 | **sy** |
| **Fisher value** | 175.552214 | 2118.000000 | **df** |
| **Regression ss** | 0.059108 | 0.713129 | **Residual ss** |

**Table 5:** LINEST value of post-1982.5 group

|  |  |
| --- | --- |
| **POST-1982.5** | |
| **σm or μm (g/year)** | 0.000051 |
| **m (g/year)** | -0.000671 |
| **m + μm (g/year)** | -0.000621 |
| **m - μm (g/year)** | -0.000722 |

**Table 6:** Comparison of mean with its standard error

|  |  |
| --- | --- |
| **POST-1982.5** | |
| **σm or μm (g/year)** | 0.000051 |
| **m (g/year)** | -0.000671 |
| **t(95% CI, df ∞)** | 1.960000 |
| **t\*μm (g/year)** | 0.000099 |
| **m + t\*μm (g/year)** | -0.000572 |
| **m - t\*μm (g/year)** | -0.000771 |

**Table 7:** Comparison of means with the standard error by student’s t-test

**Figure 3:** US One Cent coin of 1963 to 1982.0

|  |  |  |  |
| --- | --- | --- | --- |
| **LINEST PRE-1982.0** | | | |
| **m** | -0.000355 | 3.791006 | **b** |
| **μm** | 0.000136 | 0.269262 | **μb** |
| **R2** | 0.003653 | 0.030003 | **sy** |
| **Fisher value** | 6.796654 | 1854.000000 | **df** |
| **Regression ss** | 0.006118 | 1.668880 | **Residual ss** |

**Table 8:** LINEST value for pre-1982.0 group

|  |  |
| --- | --- |
| **PRE-1982.0** | |
| **σm or μm (g/year)** | 0.000136 |
| **m (g/year)** | -0.000355 |
| **m + μm (g/year)** | -0.000219 |
| **m - μm (g/year)** | -0.000492 |

**Table 9:** Comparison of means with its standard error

|  |  |
| --- | --- |
| **PRE-1982.0** | |
| **σm or μm (g/year)** | 0.000136 |
| **m (g/year)** | -0.000355 |
| **t(95% CI, df ∞)** | 1.960000 |
| **t\*μm (g/year)** | 0.000267 |
| **m + t\*μm (g/year)** | -0.000088 |
| **m - t\*μm (g/year)** | -0.000622 |

**Table 10:** Comparison of means with its standard error by student’s t-test

**YEAR-BY-YEAR ANALYSIS – F-TEST AND T-TEST**

|  |  |
| --- | --- |
| **Post-1982.5** | |
| **Highest average mass (g)** | 2.5246 |
| **Lowest average mass (g)** | 2.4924 |

**Table 11:** Highest and lowest average mass of post-1982.5 group

|  |  |  |
| --- | --- | --- |
| **Post-1982.5** | | |
| **F-Test Two-Sample for Variances** | | |
| **Year** | **1985** | **1997** |
| **Mean (g)** | 2.5246 | 2.4924 |
| **Variance (g2)** | 0.0004 | 0.0003 |
| **Observations** | 71 | 42 |
| **df** | 70 | 41 |
| **F** | 1.2400 | |
| **P(F<=f) one-tail** | 0.2303 | |
| **F Critical one-tail** | 1.6129 | |
| **F-test PASS** | | |

**Table 12:** F-test for post-1982.5 group

|  |  |  |
| --- | --- | --- |
| **Post-1982.5** | | |
| **t-Test: Two-Sample Assuming Equal Variances** | | |
| **Year** | **1985** | **1997** |
| **Mean (g)** | 2.5246 | 2.4924 |
| **Variance (g2)** | 0.0004 | 0.0003 |
| **Observations** | 71 | 42 |
| **Pooled Variance** | 0.0003 | |
| **Hypothesized Mean Difference** | 0 | |
| **df** | 111 | |
| **t Stat** | 8.9355 | |
| **P(T<=t) one-tail** | 0.0000 | |
| **t Critical one-tail** | 1.6587 | |
| **P(T<=t) two-tail** | 0.0000 | |
| **t Critical two-tail** | 1.9816 | |
| **t-test FAIL** | | |

**Table 13:** t-test of post-1982.5 group

|  |  |
| --- | --- |
| **Pre-1982.0** | |
| **Highest average mass (g)** | 3.1070 |
| **Lowest average mass (g)** | 3.0707 |

**Table 14:** Highest and lowest average mass of pre-1982.0 group

|  |  |  |
| --- | --- | --- |
| **Pre-1982.0** | | |
| **F-Test Two-Sample for Variances** | | |
| **Year** | **1971** | **1968** |
| **Mean (g)** | 3.1070 | 3.0707 |
| **Variance (g2)** | 0.0018 | 0.0008 |
| **Observations** | 66 | 57 |
| **df** | 65 | 56 |
| **F** | 2.3248 | |
| **P(F<=f) one-tail** | 0.0008 | |
| **F Critical one-tail** | 1.5395 | |
| **F-test FAIL** | | |

**Table 15:** F-test of pre-1982.0 group

|  |  |  |
| --- | --- | --- |
| **Pre-1982.0** | | |
| **t-Test: Two-Sample Assuming Unequal Variances** | | |
| **Year** | **1971** | **1968** |
| **Mean (g)** | 3.1070 | 3.0707 |
| **Variance (g2)** | 0.0018 | 0.0008 |
| **Observations** | 66 | 57 |
| **Hypothesized Mean Difference** | 0 | |
| **df** | 113 | |
| **t Stat** | 5.6190 | |
| **P(T<=t) one-tail** | 0.0000 | |
| **t Critical one-tail** | 1.6585 | |
| **P(T<=t) two-tail** | 0.0000 | |
| **t Critical two-tail** | 1.9812 | |
| **t-test FAIL** | | |

**Table 16:** t-test of pre-1982.0 group

**COMPARISON BY MINT LOCATION**

|  |  |  |
| --- | --- | --- |
| **Post-1982.5** | | |
| **Mint Location** | P | D |
| **Mean (g)** | 2.5047 | 2.5046 |
| **Standard deviation (g)** | 0.0191 | 0.0190 |
| **Minimum value (g)** | 2.4502 | 2.4480 |
| **Maximum value (g)** | 2.5616 | 2.5526 |

**Table 17:** Data for two different minted locations

|  |  |  |
| --- | --- | --- |
| **Post-1982.5** | | |
| **F-Test Two-Sample for Variances** | | |
| **Mint Location** | P | D |
| **Mean (g)** | 2.5047 | 2.5046 |
| **Variance (g2)** | 0.0004 | 0.0004 |
| **Observations** | 1967 | 153 |
| **df** | 1966 | 152 |
| **F** | 1.0080 | |
| **P(F<=f) one-tail** | 0.4868 | |
| **F Critical one-tail** | 1.2286 | |
| **F-test PASS** | | |

**Table 18:** F-test of two different minted locations for post-1982.5 group

|  |  |  |
| --- | --- | --- |
| **Post-1982.5** | | |
| **t-Test: Two-Sample Assuming Equal Variances** | | |
| **Mint Location** | P | D |
| **Mean (g)** | 2.5047 | 2.5046 |
| **Variance (g2)** | 0.0004 | 0.0004 |
| **Observations** | 1967 | 153 |
| **Pooled Variance** | 0.0004 | |
| **Hypothesized Mean Difference** | 0 | |
| **df** | 2118 | |
| **t Stat** | 0.0238 | |
| **P(T<=t) one-tail** | 0.4905 | |
| **t Critical one-tail** | 1.6456 | |
| **P(T<=t) two-tail** | 0.9810 | |
| **t Critical two-tail** | 1.9611 | |
| **t-test PASS** | | |

**Table 19:** t-test of two different minted locations for post-1982.5 group

|  |  |  |
| --- | --- | --- |
| **Pre-1982.0** | | |
| **Mint Location** | D | P |
| **Mean (g)** | 3.0933 | 3.0884 |
| **Standard deviation (g)** | 0.0327 | 0.0296 |
| **Minimum value (g)** | 2.9997 | 3.0009 |
| **Maximum value (g)** | 3.1749 | 3.1742 |

**Table 20:** Data for pre-1982.0 group

|  |  |  |
| --- | --- | --- |
| **Pre-1982.0** | | |
| **F-Test Two-Sample for Variances** | | |
| **Mint Location** | D | P |
| **Mean (g)** | 3.0933 | 3.0884 |
| **Variance (g2)** | 0.0011 | 0.0009 |
| **Observations** | 244 | 1613 |
| **df** | 243 | 1612 |
| **F** | 1.2230 | |
| **P(F<=f) one-tail** | 0.0160 | |
| **F Critical one-tail** | 1.1675 | |
| **F-test FAIL** | | |

**Table 21:** F-test for two different minted locations for pre-1982.0 group

|  |  |  |
| --- | --- | --- |
| **Pre-1982.0** | | |
| **t-Test: Two-Sample Assuming Unequal Variances** | | |
| **Mint Location** | D | P |
| **Mean (g)** | 3.0933 | 3.0884 |
| **Variance (g2)** | 0.0011 | 0.0009 |
| **Observations** | 244 | 1613 |
| **Hypothesized Mean Difference** | 0 | |
| **df** | 306 | |
| **t Stat** | 2.2064 | |
| **P(T<=t) one-tail** | 0.0141 | |
| **t Critical one-tail** | 1.6498 | |
| **P(T<=t) two-tail** | 0.0281 | |
| **t Critical two-tail** | 1.9677 | |
| **t-test PASS** | | |

**Table 22:** t-test for two different minted locations for pre-1982.0 group

**DISTRIBUTION OF MASS ANALYSIS**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Year** | **Average (g)** | **Standard Deviation (g)** | **Minimum value (g)** | **Maximum value (g)** |
| 1982.5 | 2.5149 | 0.0244 | 2.4590 | 2.5504 |
| 1983 | 2.5189 | 0.0253 | 2.4508 | 2.5616 |
| 1984 | 2.5170 | 0.0210 | 2.4710 | 2.5592 |
| 1985 | 2.5246 | 0.0192 | 2.4715 | 2.5561 |
| 1986 | 2.5192 | 0.0238 | 2.4629 | 2.5536 |
| 1987 | 2.5031 | 0.0199 | 2.4480 | 2.5562 |
| 1988 | 2.5011 | 0.0217 | 2.4506 | 2.5471 |
| 1989 | 2.5179 | 0.0190 | 2.4660 | 2.5609 |
| 1990 | 2.5076 | 0.0203 | 2.4626 | 2.5498 |
| 1991 | 2.5053 | 0.0204 | 2.4680 | 2.5492 |
| 1992 | 2.5016 | 0.0235 | 2.4502 | 2.5557 |
| 1993 | 2.5010 | 0.0152 | 2.4612 | 2.5528 |
| 1994 | 2.5027 | 0.0124 | 2.4755 | 2.5446 |
| 1995 | 2.4975 | 0.0142 | 2.4648 | 2.5563 |
| 1996 | 2.5016 | 0.0186 | 2.4644 | 2.5604 |
| 1997 | 2.4924 | 0.0173 | 2.4640 | 2.5305 |
| 1998 | 2.5024 | 0.0138 | 2.4753 | 2.5374 |
| 1999 | 2.5019 | 0.0141 | 2.4658 | 2.5458 |
| 2000 | 2.5022 | 0.0111 | 2.4688 | 2.5405 |
| 2001 | 2.5040 | 0.0144 | 2.4703 | 2.5513 |
| 2002 | 2.4996 | 0.0112 | 2.4723 | 2.5213 |
| 2003 | 2.4983 | 0.0171 | 2.4611 | 2.5603 |
| 2004 | 2.5016 | 0.0136 | 2.4686 | 2.5529 |
| 2005 | 2.5023 | 0.0197 | 2.4576 | 2.5340 |
| 2006 | 2.4993 | 0.0183 | 2.4531 | 2.5543 |
| 2007 | 2.4984 | 0.0148 | 2.4618 | 2.5323 |
| 2008 | 2.5060 | 0.0166 | 2.4766 | 2.5381 |
| 2010 | 2.4978 | 0.0140 | 2.4659 | 2.5238 |
| 2012 | 2.5155 | 0.0000 | 2.5155 | 2.5155 |
| 2013 | 2.4938 | 0.0100 | 2.4791 | 2.5083 |

**Table 23:** The data for post-1982.5 group (excluding data from 2009)

|  |  |  |
| --- | --- | --- |
| **Mass range (g)** | **Mass of penny (g)** | **Frequency** |
| **2.40-2.41** | 2.4100 | 0 |
| **2.41-2.42** | 2.4200 | 0 |
| **2.42-2.43** | 2.4300 | 0 |
| **2.43-2.44** | 2.4400 | 0 |
| **2.44-2.45** | 2.4500 | 1 |
| **2.45-2.46** | 2.4600 | 13 |
| **2.46-2.47** | 2.4700 | 47 |
| **2.47-2.48** | 2.4800 | 122 |
| **2.48-2.49** | 2.4900 | 251 |
| **2.49-2.50** | 2.5000 | 464 |
| **2.50-2.51** | 2.5100 | 472 |
| **2.51-2.52** | 2.5200 | 348 |
| **2.52-2.53** | 2.5300 | 186 |
| **2.53-2.54** | 2.5400 | 108 |
| **2.54-2.55** | 2.5500 | 67 |
| **2.55-2.56** | 2.5600 | 36 |
| **2.56-2.57** | 2.5700 | 5 |
| **2.57-2.58** | 2.5800 | 0 |
| **2.58-2.59** | 2.5900 | 0 |
| **2.59-2.60** | 2.6000 | 0 |

**Table 24:** Frequency table of the mass of pennies in the post-1982.5 group

**Figure 4:** Histogram of mass of pennies from 1982.5-2013, excluding 2009